MAC 2311
Chapter 4 Review Materials
Topics Include The Definite Integral, The Fundamental Theorem of Calculus, The Net Change Theorem and the Substitution Method

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed.

1) $f(x) = x^2$ between $x = 0$ and $x = 2$ using a lower sum with two rectangles of equal width. 1) ________

A) 4.5 B) 1 C) 5 D) 2.5

2) $f(x) = x^2$ between $x = 0$ and $x = 1$ using the "midpoint rule" with two rectangles of equal width. 2) ________

A) .3145 B) .625 C) .125 D) .75

3) $f(x) = \frac{1}{x}$ between $x = 3$ and $x = 7$ using a lower sum with two rectangles of equal width. 3) ________

A) $\frac{16}{15}$ B) $\frac{24}{35}$ C) $\frac{8}{5}$ D) $\frac{16}{35}$

4) $f(x) = \frac{1}{x}$ between $x = 2$ and $x = 5$ using an upper sum with two rectangles of equal width. 4) ________

A) $\frac{33}{70}$ B) $\frac{33}{28}$ C) $\frac{51}{28}$ D) $\frac{51}{70}$

Use a finite sum to estimate the average value of the function on the given interval by partitioning the interval and evaluating the function at the midpoints of the subintervals.

5) $f(x) = 3x^5$ on $[1, 3]$ divided into 4 subintervals 5) ________

A) 103.8 B) 703.2 C) 58.6 D) 175.8

6) $f(x) = x^2 - 8$ on $[-3, 7]$ divided into 5 subintervals 6) ________

A) 12 B) 4 C) $\frac{52}{5}$ D) 20

Write the sum without sigma notation and evaluate it.

7) $\sum_{k=1}^{2} \frac{8k}{k+17}$ 7) ________

A) $\frac{8}{1+17} + \frac{8}{2+17} = \frac{148}{171}$ B) $\frac{8}{1+17} + \frac{16}{2+17} = \frac{64}{171}$

C) $\frac{8}{1+17} + \frac{16}{2+17} = \frac{220}{171}$ D) $\frac{8}{1+17} + \frac{16}{2+17} = \frac{24}{37}$

8) $\sum_{k=1}^{3} \frac{k+8}{k}$ 8) ________

A) $\frac{1+8}{1} \cdot \frac{2+8}{2} \cdot \frac{3+8}{3} = 165$

B) $\frac{1+8}{1} + \frac{3+8}{3} = \frac{38}{3}$

C) $\frac{1+8}{1} + \frac{2+8}{2} + \frac{3+8}{3} = \frac{53}{3}$

D) $\frac{1+8}{1} + \frac{2+8}{2} + \frac{3+8}{3} = 30$

9) $\sum_{k=1}^{3} (-1)^k (k-2)^2$ 9) ________

A) $-(1-2)^2 + (2-2)^2 - (3-2)^2 = 2$

B) $-(1-2)^2 - 2(2-2)^2 - 3(3-2)^2 = -4$

C) $(1-2)^2 - (3-2)^2 = -2$

D) $-(1-2)^2 + (2-2)^2 - (3-2)^2 = -2$

10) $\sum_{k=1}^{4} \frac{k^2}{2}$ 10) ________

A) $\frac{1^2}{2} + \frac{4^2}{2} = \frac{17}{2}$

B) $\frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \frac{4^2}{2} = 15$

C) $\frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \frac{4^2}{2} = \frac{15}{2}$

D) $\frac{1^2}{2} \cdot \frac{2^2}{2} \cdot \frac{3^2}{2} \cdot \frac{4^2}{2} = \frac{567}{16}$

11) $\sum_{k=1}^{3} (-1)^k \sin\frac{3\pi}{2}$ 11) ________

A) $-\sin\frac{3\pi}{2} + \sin\frac{3\pi}{2} - \sin\frac{3\pi}{2} = -1$

B) $-\sin\frac{3\pi}{2} + \sin\frac{3\pi}{2} - \sin\frac{3\pi}{2} = 0$

C) $-\sin\frac{3\pi}{2} - \sin\frac{3\pi}{2} = 2$

D) $-\sin\frac{3\pi}{2} + \sin\frac{3\pi}{2} - \sin\frac{3\pi}{2} = 1$

Provide an appropriate response.

12) Which of the following express 1 + 5 + 25 + 125 + 625 in sigma notation? 12) ________

I. $\sum_{k=1}^{5} 5^{k-1}$ II. $\sum_{k=0}^{4} 5^k$ III. $\sum_{k=-1}^{3} 5^{k+1}$

A) I, II, and III B) I and II C) II and III D) II only

13) Which formula is not equivalent to the other two? 13) ________

I. $\sum_{k=1}^{3} \frac{(-1)^{k+1}}{k+1}$ II. $\sum_{k=-1}^{1} \frac{(-1)^{k+3}}{k+3}$ III. $\sum_{k=0}^{2} \frac{(-1)^{k-1}}{k-1}$

A) I

B) All are equivalent.

C) II

D) III

Express the sum in sigma notation.

14) 1 - 3 + 9 - 27 + 81 — 14) _______

A) $\sum_{k=-2}^{2} (-1)^{k+1} 3^{k+1}$

B) $\sum_{k=0}^{4} (-1)^{k} 3^{k}$

C) $\sum_{k=-1}^{3} (-1)^{k+1} 3^{k}$

D) $\sum_{k=1}^{5} (-3)^{k}$

15) 6 + 7 + 8 + 9 + 10 + 11 — 15) _______

A) $\sum_{k=6}^{5} k + 6$

B) $\sum_{k=-1}^{0} (-1)^{2k} k$

C) $\sum_{k=0}^{5} k + 6$

D) $\sum_{k=0}^{5} k$

16) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ — 16) _______

A) $\sum_{k=1}^{5} \left(\frac{1}{2}\right)^{k-1}$

B) $\sum_{k=0}^{4} \left(\frac{1}{2}\right)^{k+1}$

C) $\sum_{k=1}^{4} \left(\frac{1}{2}\right)^{k-1}$

D) $\sum_{k=1}^{4} \left(\frac{1}{2}\right)^{k}$

17) 5 + 10 + 15 + 20 + 25 — 17) _______

A) $\sum_{k=0}^{4} 5(k+1)$

B) $\sum_{k=2}^{5} 5(k-1)$

C) $\sum_{k=1}^{5} 5(k+1)$

D) $\sum_{k=1}^{6} 5k$

18) $-\frac{1}{7} + \frac{2}{7} - \frac{3}{7} + \frac{4}{7} - \frac{5}{7}$ — 18) _______

A) $\sum_{k=1}^{4} (-1)^{k} \frac{k+1}{7}$

B) $\sum_{k=1}^{5} (-1)^{k+1} \frac{k}{7}$

C) $\sum_{k=0}^{5} (-1)^{k-1} \frac{k}{7}$

D) $\sum_{k=1}^{5} (-1)^{k} \frac{k}{7}$

Evaluate the sum.

19) $\sum_{k=1}^{8} k^2 - 5$ — 19) _______

A) 199

B) 59

C) 204

D) 164

20) $\sum_{k=1}^{14} k$ — 20) _______

A) $\frac{105}{2}$

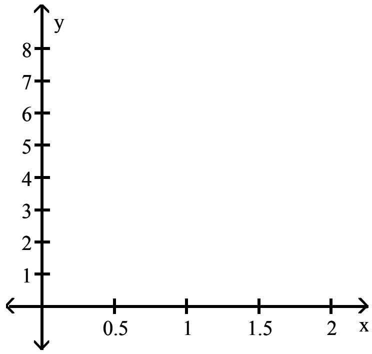
B) 105

C) 14

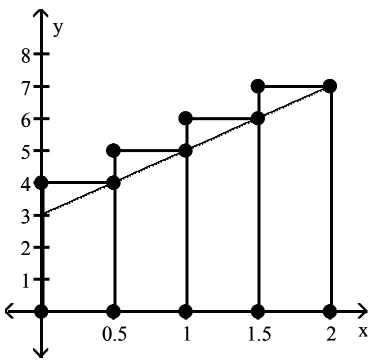
D) 210

Graph the function f(x) over the given interval. Partition the interval into 4 subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$, using the indicated point in the kth subinterval for c_k.

21) $f(x) = 2x + 3$, [0, 2], left–hand endpoint 21) _______



A)



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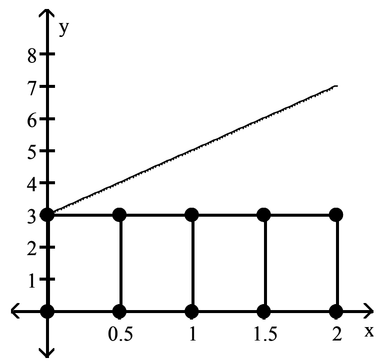

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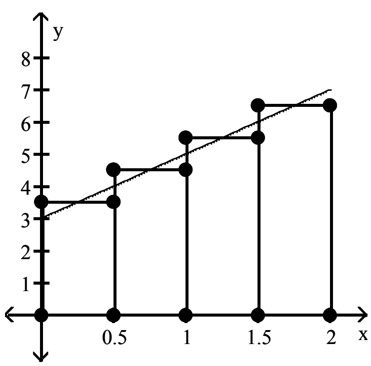

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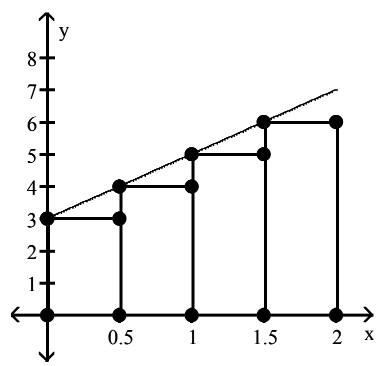
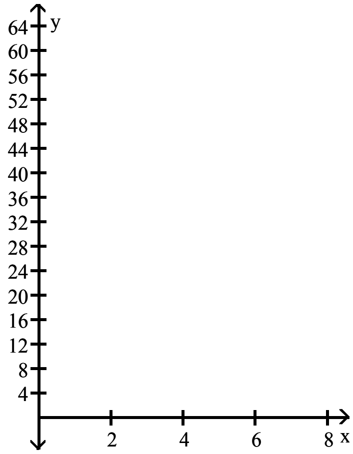
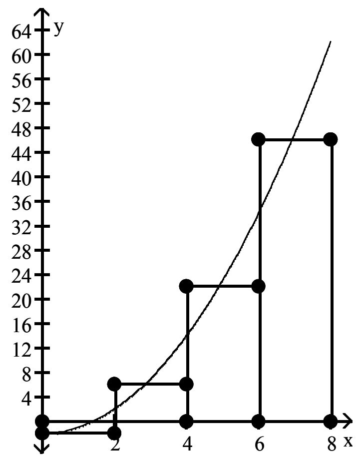


22) $f(x) = x^2 - 3$, [0, 8], midpoint

22) _______



A)



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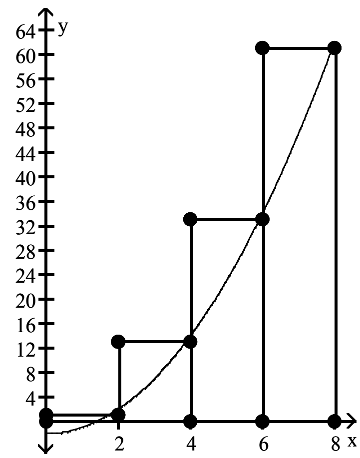

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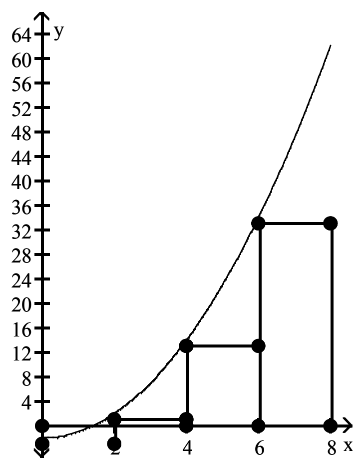

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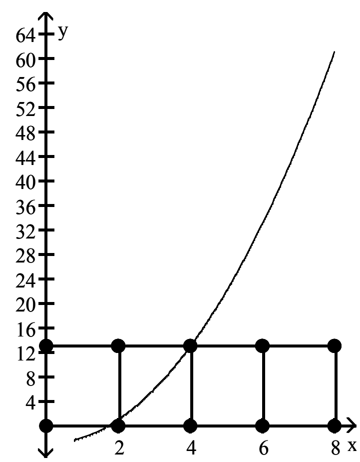
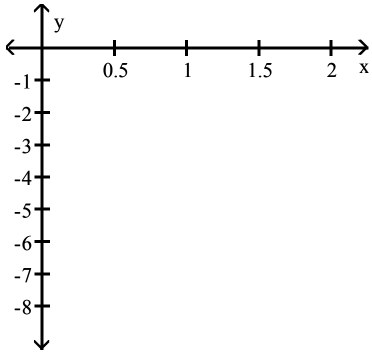
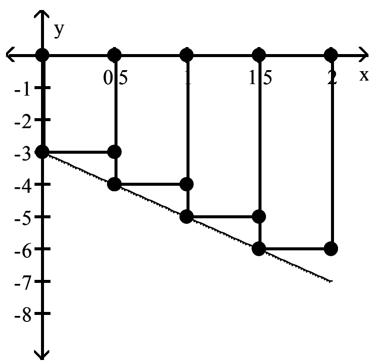


23) $f(x) = -2x - 3$, [0, 2], left–hand endpoint

23) _______



A)



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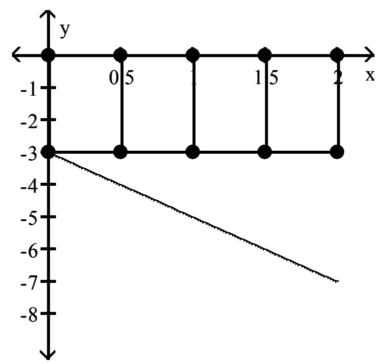

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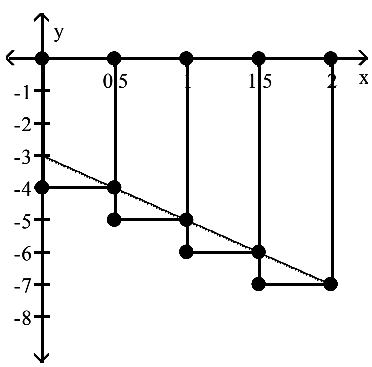

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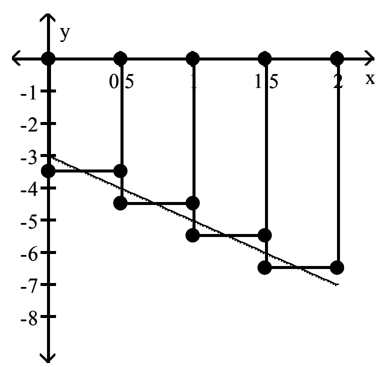
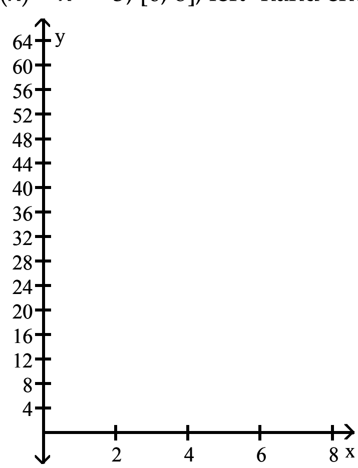
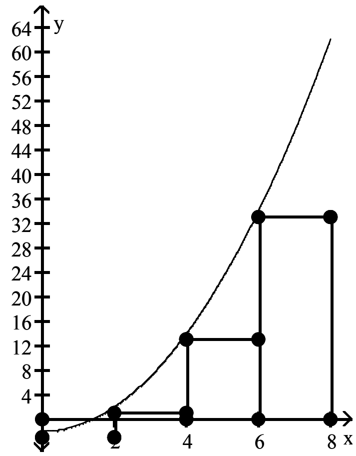


24) $f(x) = x^2 - 3$, [0, 8], left–hand endpoint

24) _______



A)



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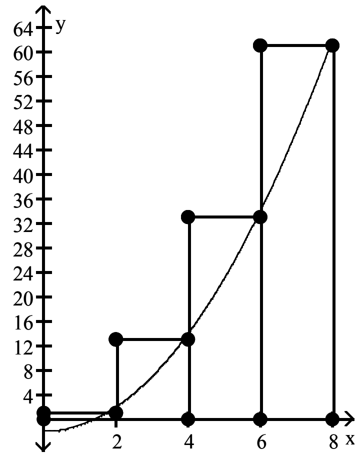

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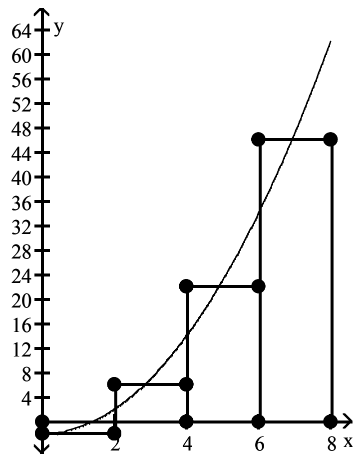
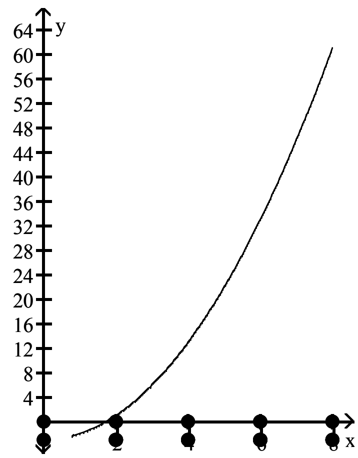
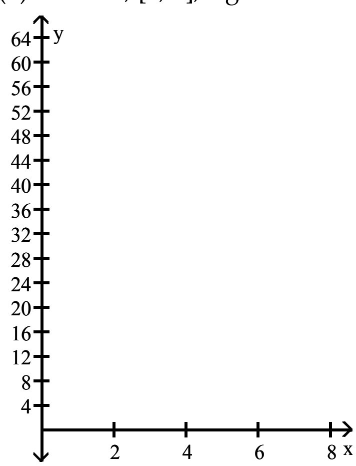


D)

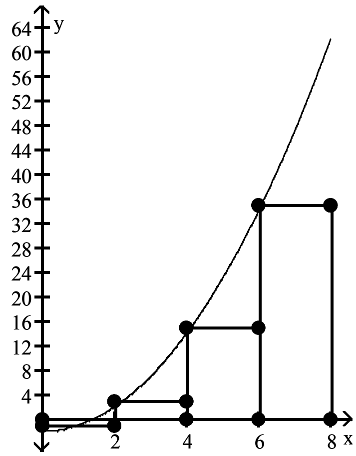


25) $f(x) = x^2 - 1$, [0, 8], right–hand endpoint

25) _______



A)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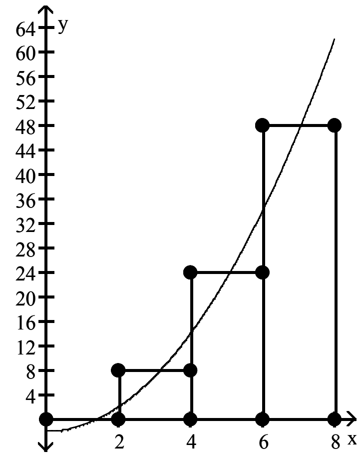
B)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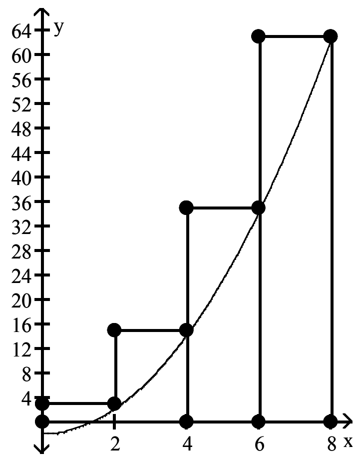
C)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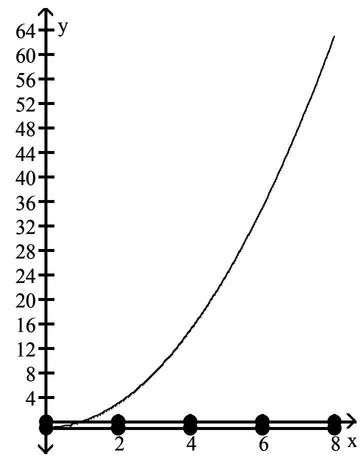
D)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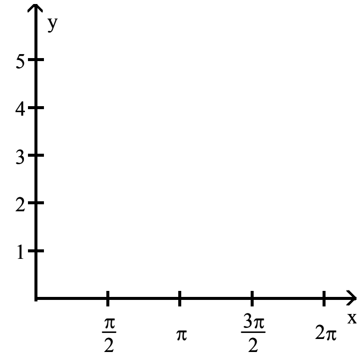
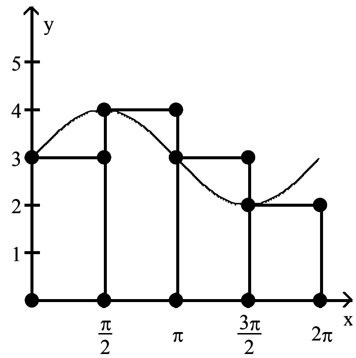
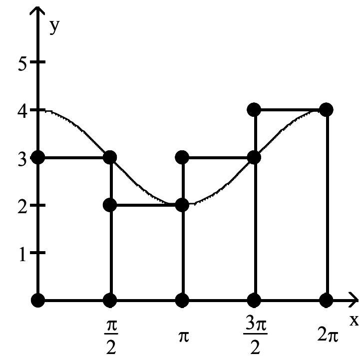
26) f(x) = cos x + 3, [0, 2π], left–hand endpoint 26) _______



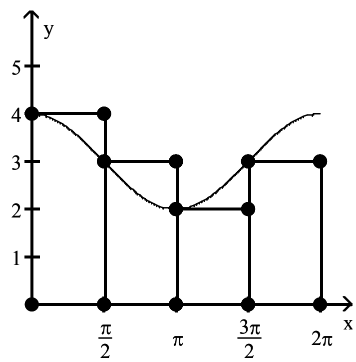
A)



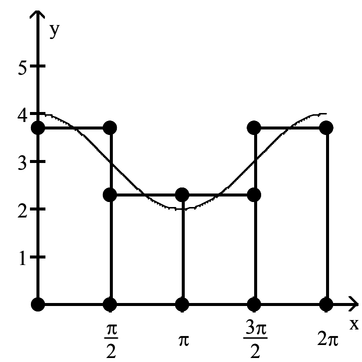
B)



C)



D)



Graph the integrand and use areas to evaluate the integral.

27) $\int_{-1}^{6} 3\, dx$ 27) _______

A) 21 B) 15 C) $\frac{21}{2}$ D) 7

28) $\int_{0}^{3} 7x\, dx$ 28) _______

A) 63 B) 21 C) $\frac{63}{2}$ D) $\frac{9}{2}$

29) $\int_{-4}^{4} \sqrt{16 - x^2}\, dx$ 29) _______

A) 4π B) 16π C) 16 D) 8π

Evaluate the integral.

30) $\int_{1}^{\sqrt{17}} x\, dx$ 30) _______

A) 8 B) -8 C) $\sqrt{17} - 1$ D) 16

31) $\int_{0}^{1/6} t^2\, dt$ 31) _______

A) $-\frac{1}{648}$ B) 648 C) $-\frac{1}{6}$ D) $\frac{1}{648}$

32) $\int_{0}^{2\pi} \theta^2\, d\theta$ 32) _______

A) $\frac{19\pi^3}{24}$ B) $\frac{7\pi^3}{3}$ C) $\frac{\pi^3}{24}$ D) $\frac{8\pi^3}{3}$

33) $\int_{0}^{6} \left(3x^2 + x + 5\right)\, dx$ 33) _______

A) 518 B) 119 C) 47 D) 264

34) $\int_{2}^{5} 5\, dx$ 34) _______

A) 10 B) 35 C) -10 D) 15

35) $\int_{0}^{16} 2\sqrt{x}\, dx$ 35) _______

A) $\frac{256}{3}$ B) 16 C) 128 D) 192

36) $\int_{0}^{3} (x + 2)^3\, dx$ 36) _______

A) 63 B) 609 C) $\frac{625}{4}$ D) $\frac{609}{4}$

37) $\int_{1}^{4} \frac{t^2 + 1}{\sqrt{t}}\, dt$ 37) _______

A) $\frac{72}{5}$ B) 32 C) $\frac{77}{5}$ D) $\frac{92}{5}$

38) $\int_{\pi/2}^{3\pi/2} 10 \cos x\, dx$ 38) _______

A) -20 B) -10 C) 20 D) 10

39) $\int_{\pi/4}^{3\pi/4} 5 \csc^2 x\, dx$ 39) _______

A) -10 B) 0 C) 5 D) 10

Find the derivative.

40) $\frac{d}{dx} \int_{0}^{x^3} \sin t\, dt$ 40) _______

A) $3x^2 \sin (x^3)$ B) $\frac{1}{4}x^4 \sin (x^3)$ C) $\sin (x^3)$ D) $-\cos (x^3) - 1$

41) $\frac{d}{dt} \int_{0}^{\sin t} \frac{1}{25 - u^2}\, du$ 41) _______

A) $\frac{1}{25 - \sin^2 t}$ B) $\frac{\cos t}{25 - \sin^2 t}$

C) $\frac{- \cos t}{25 - \sin^2 t}$ D) $\frac{1}{\cos t\,(25 - \sin^2 t)}$

42) $y = \int_{0}^{x} \sqrt{6x + 7}\, dt$ 42) _______

A) $\frac{1}{9}(6x + 7)^{3/2}$ B) $\sqrt{6x + 7}$ C) $\sqrt{6x + 7} - \sqrt{7}$ D) $\frac{3}{\sqrt{6x + 7}}$

43) $y = \int_{0}^{x^{10}} \cos \sqrt{t}\, dt$ 43) _______

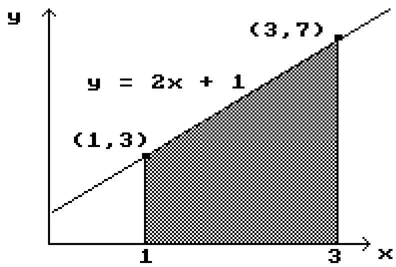
A) $\cos (x^5) - 1$ B) $10x^9 \cos (x^5)$ C) $\cos (x^5)$ D) $\sin (x^5)$

44) $y = \int_{0}^{\tan x} \sqrt{t}\, dt$ 44) _______

A) $\sqrt{\tan x}$ B) $\sec^2 x \sqrt{\tan x}$ C) $\frac{2}{3} \tan^{3/2} x$ D) $\sec x \tan^{3/2} x$

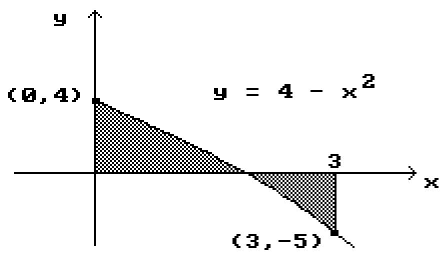
Find the area of the shaded region.

45) 45) ________



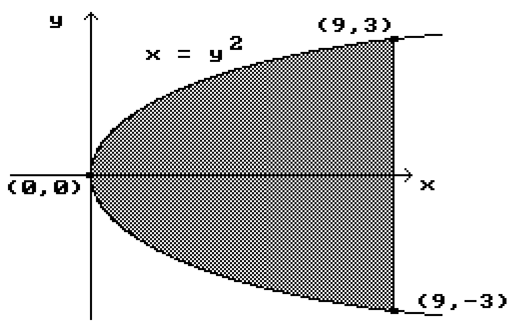
A) 12.5 B) 7.5 C) 5 D) 10

46) 46) ________



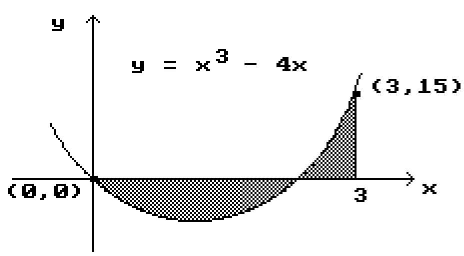
A) 3 B) $\frac{23}{3}$ C) $\frac{5}{3}$ D) 5

47) 47) ________



A) 27 B) 18 C) 45 D) 36

48) 48) ________



A) $\frac{41}{4}$ B) $\frac{33}{4}$ C) $\frac{9}{4}$ D) $\frac{17}{4}$

Solve the initial value problem.

49) $\frac{dy}{dx} = \csc x,\ \ y(3) = -5$ 49) ________

A) $y = \int_3^x \csc t\, dt - 5$

B) $y = \int_x^3 \csc t\, dt - 5$

C) $y = -\int_3^x \csc t \cot t\, dt - 5$

D) $y = \int_{-5}^x \csc t\, dt + 3$

50) $\frac{dy}{dx} = 12 \sin^2 x \cos x,\ \ y(0) = 4$ 50) ________

A) $y = -4 \sin^3 x - 4$

B) $y = 4 \sin^3 x + 4$

C) $y = 6 \cos^2 x + 4$

D) $y = 24 \sin x \cos x + 4$

Solve the problem.

51) A certain company has found that its expenditure rate per day (in hundreds of dollars) on a certain type of job is given by $\frac{dE}{dx} = 4x + 10$, where x is the number of days since the start of the job. Find the expenditure if the job takes 8 days. 51) ________

A) $20,800 B) $4200 C) $208 D) $42

52) In a certain memory experiment, subject A is able to memorize words at a rate given by 52) ________

$$\frac{dm}{dt} = -0.006t^2 + 0.4t \qquad \text{(words per minute).}$$

In the same memory experiment, subject B is able to memorize at the rate given by

$$\frac{dM}{dt} = -0.012t^2 + 0.4t \qquad \text{(words per minute).}$$

How many more words does subject B memorize from t = 0 to t = 16 (during the first 16 minutes)?

A) -8 B) 43 C) 35 D) -25

Evaluate the integral using the given substitution.

53) $\int x \cos (5x^2)\ dx,\quad u = 5x^2$ 53) ________

A) $\frac{1}{10} \sin (5x^2) + C$

B) $\frac{1}{u} \sin (u) + C$

C) $\frac{x^2}{2} \sin (5x^2) + C$

D) $\sin(5x^2) + C$

54) $\int \left(1 - \sin\frac{t}{4}\right)^2 \cos\frac{t}{4}\ dt,\quad u = 1 - \sin\frac{t}{4}$ 54) ________

A) $-\frac{4}{3}\left(1 - \sin\frac{t}{4}\right)^3 + C$

B) $4\left(1 - \sin\frac{t}{4}\right)^3 + C$

C) $\frac{1}{3}\left(1 - \sin\frac{t}{4}\right)^3 \sin\frac{t}{4} + C$

D) $\frac{4}{3}\left(1 - \cos\frac{t}{4}\right)^3 + C$

55) $\int 18(6x - 7)^{-3}\ dx,\quad u = 6x - 7$ 55) ________

A) $-3(6x - 7)^{-2} + C$

B) $(6x - 7)^{-2} + C$

C) $-\frac{3}{2}(6x - 7)^{-2} + C$

D) $-\frac{3}{4}(6x - 7)^{-4} + C$

56) $\int x^4(x^5 - 7)^4\ dx,\quad u = x^5 - 7$ 56) ________

A) $\frac{1}{15}(x^5 - 7)^3 + C$

B) $\frac{1}{25}(x^5 - 7)^5 + C$

C) $\frac{1}{5}(x^5 - 7)^5 + C$

D) $\frac{1}{25}x^{25} - 7 + C$

57) $\int \frac{4s^3\ ds}{\sqrt{2 - s^4}},\quad u = 2 - s^4$ 57) ________

A) $-2s^3\sqrt{2 - s^4} + C$

B) $-2\sqrt{2 - s^4} + C$

C) $\frac{-1}{2\sqrt{2 - s^4}} + C$

D) $\frac{2s^4}{\sqrt{2 - s^4}}$

58) $\int 18(y^6 + 4y^3 + 6)^3(2y^5 + 4y^2)\ dy,\quad u = y^6 + 4y^3 + 6$ 58) ________

A) $18(y^6 + 4y^3 + 6)^2 + C$

B) $\frac{3}{2}(y^6 + 4y^3 + 6)^4 + C$

C) $\frac{9}{2}(y^6 + 4y^3 + 6)^4 + C$

D) $\frac{9}{2}(y^6 + 4y^3 + 6)^4(10y^4 + 8y) + C$

59) $\int \sqrt{x} \cos^2 (x^{3/2} - 8)\ dx,\quad u = x^{3/2} - 8$ 59) ________

A) $x^{3/2} - 8 + \frac{1}{2} \sin 2(x^{3/2} - 8) + C$

B) $\frac{2}{9}\sin^3 (x^{3/2} - 8) + C$

C) $\frac{1}{3}\left(\sqrt{x}\right) \sin (x^{3/2} - 8) + C$

D) $\frac{1}{3}(x^{3/2} - 8) + \frac{1}{6} \sin 2(x^{3/2} - 8) + C$

60) $\int \frac{5}{x^2}\sin^2\left(\frac{5}{x}\right)\,dx$, $u = -\frac{5}{x}$ 60) ________

A) $-\frac{5}{2x} + \frac{1}{4}\sin\frac{10}{x} + C$

B) $\frac{5}{2x} + \frac{1}{2}\sin\frac{5}{x} + C$

C) $-\frac{5}{x} + \sin^3\frac{10}{x} + C$

D) $-\frac{5}{x} + \frac{1}{2}\sin\frac{10}{x} + C$

61) $\int \csc^2 9\theta \cot 9\theta\ d\theta$, $u = \cot 9\theta$ 61) ________

A) $-\frac{1}{18}\tan^2 9\theta + C$

B) $-\frac{1}{18}\cot^2 9\theta + C$

C) $\frac{1}{6}\csc^3 9\theta \cot^2 9\theta + C$

D) $\frac{1}{18}\cot^2 \theta + C$

62) $\int \frac{dx}{\sqrt{6x+1}}$, $u = 6x + 1$ 62) ________

A) $2\sqrt{6x+1} + C$

B) $\frac{1}{6(6x+1)^{3/2}} + C$

C) $3\frac{1}{\sqrt{6x+1}} + C$

D) $\frac{1}{3}\sqrt{6x+1} + C$

Evaluate the integral.

63) $\int \frac{x\,dx}{(7x^2+3)^5}$ 63) ________

A) $-\frac{7}{3}(7x^2+3)^{-4} + C$

B) $-\frac{1}{56}(7x^2+3)^{-4} + C$

C) $-\frac{7}{3}(7x^2+3)^{-6} + C$

D) $-\frac{1}{14}(7x^2+3)^{-6} + C$

64) $\int x^5\sqrt{x^6+5}\,dx$ 64) ________

A) $\frac{1}{9}\left(x^6+5\right)^{3/2} + C$

B) $\frac{2}{3}\left(x^6+5\right)^{3/2} + C$

C) $4\left(x^6+5\right)^{3/2} + C$

D) $-\frac{1}{3}\left(x^6+5\right)^{-1/2} + C$

65) $\int 10x^2\sqrt[4]{2+3x^3}\,dx$ 65) ________

A) $\frac{8}{9}\left(2+3x^3\right)^{5/4} + C$

B) $-\frac{20}{3}\left(2+3x^3\right)^{-3/4} + C$

C) $8\left(2+3x^3\right)^{5/4} + C$

D) $10\left(2+3x^3\right)^{5/4} + C$

66) $\int \csc^2 (5\theta + 3)\, d\theta$ 66) ________

A) $10 \csc (5\theta + 3) \cot (5\theta + 3) + C$

B) $-\cot (5\theta + 3) + C$

C) $-\frac{1}{5} \cot (5\theta + 3) + C$

D) $5 \cot (5\theta + 3) + C$

67) $\int \frac{\sin t}{(3 + \cos t)^5}\, dt$ 67) ________

A) $\frac{1}{6(3 + \cos t)^6} + C$

B) $\frac{1}{(3 + \cos t)^4} + C$

C) $\frac{1}{4(3 + \cos t)^4} + C$

D) $\frac{4}{(3 + \cos t)^4} + C$

68) $\int \frac{1}{t^2} \sin\left(\frac{3}{t} + 3\right) dt$ 68) ________

A) $3 \cos\left(\frac{3}{t} + 3\right) + C$

B) $\frac{1}{3} \cos\left(\frac{3}{t} + 3\right) + C$

C) $-\frac{1}{3} \cos\left(\frac{3}{t} + 3\right) + C$

D) $-\cos\left(\frac{3}{t} + 3\right) + C$

Solve the problem.

69) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 69) ________

$v = -15t + 3,\ s(0) = 14$

A) $s = -\frac{15}{2}t^2 + 3t - 14$

B) $s = -15t^2 + 3t + 14$

C) $s = \frac{15}{2}t^2 + 3t - 14$

D) $s = -\frac{15}{2}t^2 + 3t + 14$

70) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 70) ________

$v = \frac{8}{\pi} \sin \frac{4t}{\pi},\ s(\pi^2) = 2$

A) $s = 2 \cos \frac{4t}{\pi} + 4$

B) $s = -2 \cos \frac{4t}{\pi} + 8$

C) $s = -2 \cos \frac{4t}{\pi} + 3$

D) $s = -2 \cos \frac{4t}{\pi} + 4$

71) Given the acceleration, initial velocity, and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 71) ________

$a = 10 \cos 4t,\ v(0) = -7,\ s(0) = -3$

A) $s = -\frac{5}{8} \sin 4t - 7t - 3$

B) $s = \frac{5}{8} \sin 4t - 7t - 3$

C) $s = \frac{5}{8} \cos 4t + 7t - 3$

D) $s = -\frac{5}{8} \cos 4t - 7t - 3$

1) B
2) A
3) B
4) B
5) D
6) B
7) C
8) C
9) D
10) B
11) D
12) A
13) D
14) B
15) C
16) D
17) A
18) D
19) D
20) B
21) D
22) A
23) A
24) A
25) C
26) C
27) A
28) C
29) D
30) A
31) D
32) D
33) D
34) D
35) A
36) D
37) A
38) A
39) D
40) A
41) B
42) B
43) B
44) B
45) D
46) B
47) D
48) A
49) A
50) B

51) A
52) A
53) A
54) A
55) C
56) B
57) B
58) B
59) D
60) A
61) B
62) D
63) B
64) A
65) A
66) C
67) C
68) B
69) D
70) D
71) D